

Interplay of Pomeranchuk instability and superconductivity in the two-dimensional repulsive Hubbard model

Motoharu Kitatani,¹ Naoto Tsuji,² and Hideo Aoki^{1,3}

¹*Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan*

²*RIKEN Center for Emergent Matter Science (CEMS), Wako 351-0198, Japan*

³*Electronics and Photonics Research Institute, Advanced Industrial Science and Technology (AIST), Tsukuba, Ibaraki 305-8568, Japan*

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Interplay of Pomeranchuk instability (spontaneous symmetry breaking of the Fermi surface) and d -wave superconductivity is studied for the repulsive Hubbard model on the square lattice with the dynamical mean field theory combined with the fluctuation exchange approximation (FLEX+DMFT). We show that the four-fold symmetric Fermi surface becomes unstable against a spontaneous distortion into two-fold near the van Hove filling, where the symmetry of superconductivity coexisting with the Pomeranchuk distorted Fermi surface is modified from the d -wave pairing to $(d+s)$ -wave. By systematically shifting the position of van Hove filling with varied second- and third-neighbor hoppings, we find that the transition temperature T_c^{PI} of Pomeranchuk instability is more sensitively affected by the position of van Hove filling than the superconducting T_c^{SC} . This implies that the filling region for strong Pomeranchuk instability and that for strong superconducting fluctuations can be separated, and Pomeranchuk instability can appear even if the peak of T_c^{PI} is lower than the peak of T_c^{SC} . An interesting observation is that the Fermi surface distortion can enhance the superconducting T_c^{SC} in the overdoped regime, which is explained with a perturbation picture for small distortions.

I. INTRODUCTION

High- T_c cuprate superconductors harbor many fundamental questions, which challenge elaborate numerical analysis on superconductivity, magnetism and other properties. Specifically, there is growing realization that various instabilities can exist along with superconductivity¹, and the relation between various charge instabilities and superconductivity in the cuprates is now being intensively studied^{2–5}. Also, some experiments suggest a spontaneous breakdown of the four-fold symmetry of electronic states in the tetragonally structured cuprates, which is viewed as a kind of “electronic nematicity”^{6,7}. There are some explanations for the nematicity, e.g. in the context of fluctuating stripe orders⁸. Pomeranchuk instability, a spontaneous breaking of four-fold symmetry of the Fermi surface without lattice distortion, is evoked as another possible candidate for nematicity in cuprate superconductors⁹.

The presence of Pomeranchuk instability in two-dimensional lattice models has been suggested in Refs. 10,11, where the forward scattering was found to develop to induce Pomeranchuk instability. Subsequently, properties of this instability were studied primarily in mean-field models (“ f -model”), where the electrons interact only via forward scattering^{12,13}. For the two-dimensional (2D) Hubbard model on the square lattice, a representative model for cuprates, the existence of this instability is yet to be fully clarified microscopically. Functional renormalization group (fRG) calculations suggest that the superconducting fluctuation is stronger than Pomeranchuk instability¹⁴, while other numerical renormalization-group approaches suggest Pomeranchuk instability to be stronger around van Hove fillings¹⁵.

Gutzwiller wave functions combined with an efficient diagrammatic expansion technique (DE-GWF) obtained a ground state with a coexistence of the nematic order and superconductivity in 2D Hubbard model¹⁶. Also, the dynamical cluster approximation (DCA) and cellular dynamical-mean-field theory (CDMFT) showed large responses against small distortions of the lattice^{17,18}, from which the possibility of spontaneous symmetry breaking is suggested to occur at lower temperatures or for larger cluster sizes. While these results suggest that the 2D repulsive Hubbard model has a strong tendency toward the Pomeranchuk instability, whether or not this instability has higher transition temperature (T_c^{PI}) than that of superconductivity (T_c^{SC}) has to be elaborated. More importantly, the relation between the Pomeranchuk instability and superconductivity (e.g., whether they are cooperative or competing) is an intriguing question. While the mean-field study for the phenomenological model suggests that they are competing with each other with T_c^{SC} suppressed in the coexistence region¹⁹, the relation should be clarified by going beyond mean-field approaches.

Given the situation, we study in the present paper superconductivity and Pomeranchuk instability in the intermediate correlation regime by evoking FLEX+DMFT^{20,21}, a diagrammatic extension of the dynamical mean field theory (DMFT), which takes into account the spin and charge fluctuation effects on top of the DMFT local self-energy and can reproduce the dome structure in T_c^{SC} ²¹. The advantages of this method are first, there is no finite-size effect in contrast to DCA and CDMFT, which should be important for capturing small Fermi surface deformations, and second, we can calculate finite-temperature regions in contrast DE-GWF to cap-

ture the effect of this nematicity on the superconducting T_c^{SC} . This also enables us to systematically examine the relation between superconductivity and Pomeranchuk instability when the electron band filling and the second and further neighbor hoppings (t', t'') are varied. After confirming the existence of Pomeranchuk instability near the van Hove filling consistently with the previous works, we shall study the superconducting phase, which indicates that the symmetry of the gap function is changed from the ordinary d -wave pairing to $(d + s)$ -wave^{16,17}. Interestingly, the Fermi surface distortion can enhance the superconductivity in the overdoped (or strongly frustrated) regime with larger t', t'' . We shall explain this T_c^{SC} enhancement by a perturbation picture for small Fermi surface distortions, and also by the random phase approximation (RPA) in the weak-coupling regime.

We also reveal that the Pomeranchuk instability temperature T_c^{PI} is more sensitive to (t', t'') , hence the Fermi surface warping, than the superconducting T_c^{SC} . This contrasts with the previous mean-field calculations¹⁹ that showed almost the same filling dependence for the two transition temperatures. This should come from the fact that the present formalism takes account of the filling dependence of the pairing interaction beyond mean-field levels. The result also implies that the region for stronger superconducting fluctuations and that for stronger Pomeranchuk instability can be separated.

II. FORMULATION

We consider the standard repulsive Hubbard model on the square lattice with a Hamiltonian,

$$\mathcal{H} = \sum_{\mathbf{k}, \sigma} \epsilon(\mathbf{k}) c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma} + U \sum_i n_{i, \uparrow} n_{i, \downarrow}, \quad (1)$$

where $c_{\mathbf{k}, \sigma}^\dagger$ creates an electron with wave-vector $\mathbf{k} = (k_x, k_y)$ and spin σ , U is the on-site Coulomb interaction, and $n_{i, \sigma} = c_{i, \sigma}^\dagger c_{i, \sigma}$. In the presence of second-neighbor (t') and third-neighbor (t'') hopping parameters, the 2D band dispersion is given as

$$\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos 2k_x + \cos 2k_y) - \mu, \quad (2)$$

where t is the nearest-neighbor hopping (the unit of energy hereafter), and μ the chemical potential. We basically adopt $t' = -0.20t$, $t'' = 0.16t$, which are determined to fit the band calculation for a typical hole-doped single-layer cuprate, $\text{HgBa}_2\text{CuO}_{4+\delta}$ ^{22,23}.

For the numerical procedure, we employ FLEX+DMFT method, which is a kind of diagrammatic extension of DMFT, where the fluctuation exchange approximation (FLEX) and the DMFT are combined, with a double self-consistency loop. This kind of scheme has been considered in Ref.^{20,24}, and has recently been formulated through Luttinger-Ward functional with

applications to superconducting states in Ref.²¹. The latter can describe a T_c^{SC} dome against the band filling along with a spectral weight transfer. These are a virtue of FLEX+DMFT that corrects the overestimated local-FLEX self-energy in a filling-dependent manner. In FLEX+DMFT, the self-energy is calculated through the FLEX self-energy and DMFT self-energy as

$$\Sigma(k) = \Sigma_{\text{FLEX}}(k) - \Sigma_{\text{FLEX}}^{\text{loc}}(k) + \Sigma_{\text{imp}}(\omega_n). \quad (3)$$

The FLEX self-energy $\Sigma_{\text{FLEX}}(k)$ is given as

$$\Sigma_{\text{FLEX}}(k) = \frac{1}{N_k \beta} \sum_{k'} \left[\frac{3}{2} U^2 \frac{\chi_0(k - k')}{1 - U \chi_0(k - k')} + \frac{1}{2} U^2 \frac{\chi_0(k - k')}{1 + U \chi_0(k - k')} - U^2 \chi_0(k - k') \right] G(k'). \quad (4)$$

Here N_k and β are the number of k -points and inverse temperature, respectively, $k \equiv (\omega_n, \mathbf{k})$ with ω_n the Matsubara frequency for fermions, $G(k)$ Green's function, and

$$\chi_0(q) = -\frac{1}{N_k \beta} \sum_k G(k + q) G(k) \quad (5)$$

is the irreducible susceptibility. The local part of the FLEX self-energy, $\Sigma_{\text{FLEX}}^{\text{loc}}$, is computed by replacing Green's function G with the local one, $G_{\text{loc}} \equiv (1/N_k) \sum_{\mathbf{k}} G(k)$, in Eqs. (4) and (5).

For calculating the DMFT self-energy, Σ_{imp} , we need to solve the impurity problem in DMFT. Here, we employ the modified iterative perturbation theory (modified IPT) as an impurity solver. In this method, the original IPT is modified for systems without particle-hole symmetry²⁵, thus applicable to frustrated or non-half-filled cases. This is not computationally expensive, which enables us to scan over various parameter regions. We have checked, by using ALPS library^{28,29}, that the continuous-time quantum Monte Carlo (CT-QMC) impurity solver^{26,27} gives basically similar results even away from half-filling in the intermediate-coupling regime [see Fig. 1(a)].

After obtaining Green's function, we plug it into the linearized Eliashberg equation,

$$\lambda \Delta(k) = -\frac{1}{N_k \beta} \sum_{k'} V_{\text{eff}}(k - k') |G(k')|^2 \Delta(k'), \quad (6)$$

where $\Delta(k)$ is the anomalous self-energy, while

$$V_{\text{eff}}(k) = U + \frac{3}{2} U^2 \frac{\chi_0(k)}{1 - U \chi_0(k)} - \frac{1}{2} U^2 \frac{\chi_0(k)}{1 + U \chi_0(k)} \quad (7)$$

is the effective pairing interaction, and λ the eigenvalue of Eliashberg's equation. Superconducting T_c^{SC} is determined as the temperature at which $\lambda = 1$. In the right hand side of Eq. (6), we have neglected the local DMFT vertex contribution, whose validity is discussed in Appendix A.

To allow the Pomeranchuk instability to occur, we introduce a seed to deform the Fermi surface in the initial input for Green's function. While we linearize the anomalous part of Green's function as being infinitesimal, we can deal with finite Pomeranchuk order parameters, so that we can discuss superconductivity for finite distortions in this formalism.

III. RESULTS

A. Pomeranchuk instability

The Pomeranchuk order parameter η can be defined, for the originally four-fold cosine bands, as

$$\eta = \sum_{\mathbf{k}} (\cos k_x - \cos k_y) \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \rangle, \quad (8)$$

and we display the result against temperature and band filling in Fig. 1(a),(b), respectively. We can see that the order parameter starts to grow continuously with decreasing temperature, which indicates a second-order phase transition. If we turn to the filling dependence, we observe the order parameter abruptly growing around the edges of the Pomeranchuk phase, indicative of transferring to the first-order phase transition consistently with the previous work¹². Hereafter, we focus on the filling region around the peak of T_c^{PI} , where the transition is second order. The peak sits at about $n = 0.66$ for the value of the parameter considered ($U/t = 4.0$).

If we look at the Fermi surface in Fig. 1(c) for $U/t = 4.0$, $n = 0.66$, we can see that the Fermi surface, identified as the ridges in the spectral function $A(\mathbf{k}, \omega = 0)$, indeed becomes distorted at lower temperatures, $T < T_c^{\text{PI}}$.

The phase diagram is displayed in Fig. 2(a), where we can see that the Pomeranchuk instability temperature is peaked around $n = 0.66$ for the present parameter set. A yellow arrow indicates the van Hove filling in the interacting system at which the spectrum is peaked at the Fermi energy, see Fig. 2(b), for which we have obtained the spectral function with the Padé approximation, and confirmed the peak position does not change for $T > T_c^{\text{PI}}$. The fact that the Pomeranchuk instability tends to be strong around this filling is consistent with the previous results^{12,19}. The peak in the Pomeranchuk dome does not precisely coincide with the van Hove filling, which may be an effect of the asymmetric density of states¹² as in Fig. 2(b). By contrast, the present result shows that the superconducting T_c^{SC} is a monotonically decreasing function of the hole doping near the van Hove filling.

In the previous mean-field calculations¹⁹ which ignore the filling dependence of the effective pairing interaction, both T_c^{SC} and T_c^{PI} are peaked around the van Hove filling. Thus the present result indicates that the filling dependence of the effective interaction has an important effect of rendering distinction of optimal doping levels between Pomeranchuk T_c^{PI} dome and superconducting T_c^{SC}

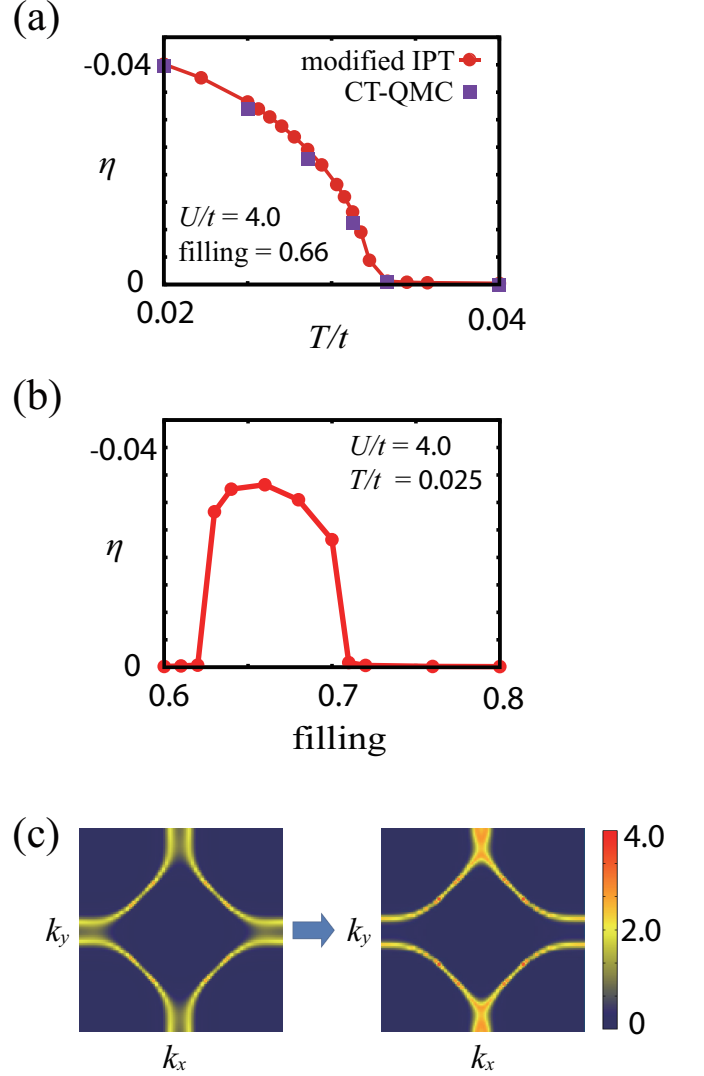


FIG. 1: (Color online) (a) Temperature dependence and (b) filling dependence of the Pomeranchuk order parameter η for $U/t = 4.0$, $(t', t'') = (-0.20, 0.16)$. In (a), the circles and squares correspond to the results of FLEX+DMFT using the modified IPT and CT-QMC as a DMFT impurity solver, respectively. (c) Fermi surface [as represented by the color-coded spectral weight $A(\mathbf{k}, \omega = 0)$] with $n = 0.66$, $U/t = 4.0$, $(t', t'') = (-0.20, 0.16)$, for $T = 0.0333t > T_c^{\text{PI}}$ ($\beta t = 30$; left) and $T = 0.0286t < T_c^{\text{PI}}$ ($\beta t = 35$; right).

dome. To confirm this difference, let us systematically vary the second- and third-neighbor hopping parameters (t', t'') in Fig. 3, which change the Fermi surface warping as well as the van Hove filling. We can see that, for a fixed $n = 0.80$, the change in the parameters shifts the distance of the filling from the van Hove filling as represented by the blurring of the spectral function around $(0, \pi)$, $(\pi, 0)$. The left of Fig. 3 plots the phase diagrams for three typical cases with different Fermi surface warping. We find that the Pomeranchuk T_c^{PI} drastically changes along with the van Hove filling (yellow allows), while the supercon-

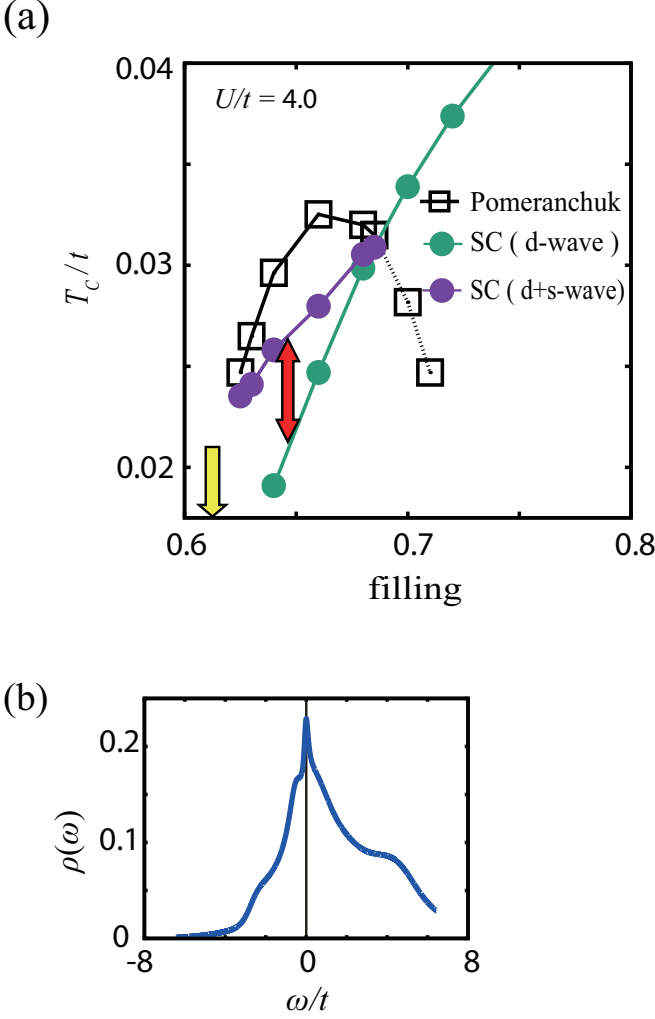


FIG. 2: (Color online) (a) Phase diagram against temperature T/t and band filling n for $U/t = 4.0$, $(t', t'') = (-0.20, 0.16)$. We show the superconducting T_c^{SC} with undistorted Fermi surface (green circles), superconducting T_c^{SC} with Fermi surface distortion (purple circles), and Pomeranchuk T_c^{PI} (black squares). The dotted line represents T_c^{PI} when we ignore the superconductivity. The yellow arrow indicates the van Hove filling in the interacting system. (b) Density of states at the filling indicated by the yellow arrow in (a) for $\beta t = 20$, $U/t = 4.0$, $(t', t'') = (-0.20, 0.16)$.

ducting T_c^{SC} is much less sensitive.

We can thus conclude that, despite both of superconductivity and Pomeranchuk instabilities being Fermi surface instabilities affected by the spectral weight at the Fermi energy, the Pomeranchuk instability is much more sensitive to the Fermi surface shape (distance from the van Hove filling). This implies that we can separate the dominant regions for the two instabilities by changing the position against the van Hove filling (dominated by t', t'').

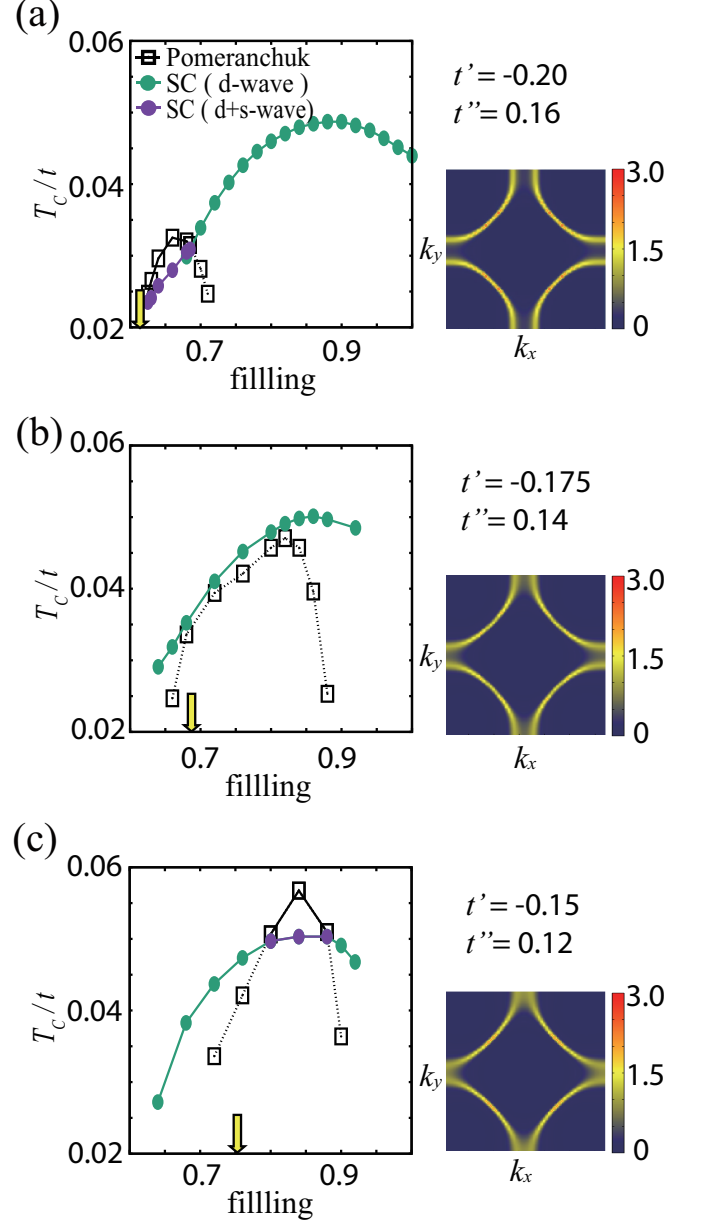


FIG. 3: (Color online) The superconducting and Pomeranchuk phase boundaries for $U/t = 4.0$ (left panels) and the spectral weight $A(\mathbf{k}, \omega = 0)$ for $n = 0.80$, $\beta t = 20$, $U/t = 4.0$ (right) are shown for $(t', t'') = (-0.20, 0.16)$ (a), $(-0.175, 0.14)$ (b), and $(-0.15, 0.12)$ (c). The symbols are the same as in Fig. 2(a), and yellow arrows indicate respective van Hove fillings in the interacting system.

B. Superconductivity under Fermi surface distortions

Now, an intriguing issue is how superconductivity behaves in the presence of the Pomeranchuk Fermi surface distortion. We actually plot in Fig. 4(a) the superconducting order parameter, where the pairing symmetry is seen to be distorted from the ordinary d -wave

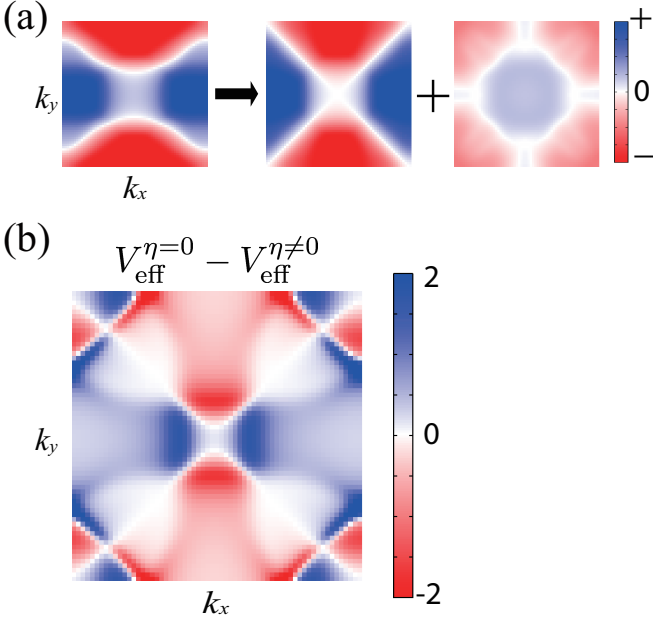


FIG. 4: (Color online) (a) Momentum dependence of the gap function for $T = 0.0286t < T_c^{\text{PI}}$ with $n = 0.66, U/t = 4.0, (t', t'') = (-0.20, 0.16)$ (left panel), which can be decomposed into a d -wave part and an (extended) s -wave (four-fold symmetric) part (right). (b) Difference in the pairing interaction with the Fermi surface distortion ($V_{\text{eff}}^{\eta \neq 0}$) and without ($V_{\text{eff}}^{\eta = 0}$), for $n = 0.66, \beta t = 31, U/t = 4.0, (t', t'') = (-0.20, 0.16)$.

to d +(extended) s -wave. Here, an interesting observation is that the superconducting T_c^{SC} can be enhanced by the Pomeranchuk distortion of the Fermi surface. Indeed, if we go back to Fig. 2, we have also plotted the superconducting T_c^{SC} when the four-fold Fermi surface is artificially imposed below Pomeranchuk T_c^{PI} (green dots). We can see the T_c^{SC} with the distorted Fermi surface (purple dots) is higher.

To identify the origin of this enhancement, we can go back to the pairing interaction to compare between the cases of Pomeranchuk-distorted and the four-fold-imposed Fermi surfaces. Figure 4(b) plots the difference of the two cases for the same parameters ($U/t = 4.0, n = 0.66$ and $\beta t = 31$). We can see that the Pomeranchuk instability distorts the pairing interaction, with a d -wave-like sign reversal.

To pin-point the origin of the distortion effect on the superconducting T_c^{SC} , we can consider the perturbational effect for small distortions, based on a general linearized gap equation,

$$\lambda \phi(k) = -\frac{1}{N_k \beta} \sum_{k'} K(k, k') \phi(k'), \quad (9)$$

where $\phi(k) = |G(k)|\Delta(k)$, while $K(k, k')$ is the kernel, given as $K(k, k') = |G(k)|V_{\text{eff}}(k - k')|G(k')|$ in FLEX+DMFT (as seen by multiplying $|G|$ to both sides

in Eq. (6)). If we consider small d -wave-like distortions [as displayed in Fig. 4(b)] for this kernel,

$$K(k, k') \rightarrow K(k, k') + \delta K^d(k, k'), \quad (10)$$

the first-order perturbation for the maximum eigenvalue λ_{max} satisfies

$$\delta \lambda_{\text{max}}^{(1)} = \sum_{k, k'} \phi_{\text{max}}^*(k) \delta K^d(k, k') \phi_{\text{max}}(k') = 0, \quad (11)$$

where the ϕ_{max} is the eigenvector for the λ_{max} , and $\delta \lambda_{\text{max}}^{(1)}$ identically vanishes due to the d -wave nature of the δK^d , so that the leading term is the second-order one,

$$\delta \lambda_{\text{max}}^{(2)} = \sum_{i, k, k'} \frac{|\phi_{\text{max}}^*(k) \delta K^d(k, k') \phi_i(k')|^2}{\lambda_{\text{max}} - \lambda_i} > 0, \quad (12)$$

where i is the index for the eigenvalue λ_i and eigenvector ϕ_i of the kernel matrix K . Since this expression is positive-definite, small d -wave deformations of the kernel in the linearized gap equation always enhance the superconducting T_c^{SC} . This explains the T_c^{SC} enhancement in Fig. 2(a), and can provide a new pathway for enhancing superconducting T_c^{SC} in terms of Fermi surface distortion.

However, it should be difficult to achieve purely d -wave like distortions for the kernel, and the terms having some other symmetries should in general arise even from purely d -wave distortions of the Fermi surface. We can elaborate this by introducing a parameter g_k where g_k represents either (i) spontaneous distortion of the electronic states ($\delta g_k = G(k) - G_{\text{undistorted}}(k)$) or (ii) small d -wave modulation of the Hamiltonian ($\delta \mathcal{H} = \sum_{\mathbf{k}, \sigma} \delta g_k c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma}$). Then we can expand the interaction kernel in g_k , which gives, up to the second-order,

$$K(k, k') \rightarrow K(k, k') + \sum_p \frac{\delta K}{\delta g_p} \delta g_p + \frac{1}{2} \sum_{p, q} \frac{\delta^2 K}{\delta g_p \delta g_q} \delta g_p \delta g_q, \quad (13)$$

and the effect on the eigenvalue λ reads

$$\delta \lambda_{\text{max}}^{(2)} = \sum_{i, k, k'} \frac{|\phi_{\text{max}}^*(k) \sum_p \frac{\delta K}{\delta g_p} \delta g_p \phi_i(k')|^2}{\lambda_{\text{max}} - \lambda_i} - \sum_{k, k'} \phi^*(k) \frac{1}{2} \sum_{p, q} \frac{\delta^2 K}{\delta g_p \delta g_q} \delta g_p \delta g_q \phi(k'). \quad (14)$$

We can see that whether T_c^{SC} can be enhanced depends on the second term on the right-hand side of Eq. (14). From this we expect that the enhancement tends to occur when the second-largest eigenvalue is close to the largest one, for which the first term on the right-hand side of Eq. (14) becomes dominant.

We can in fact check this argument in the weak-coupling case. To obtain qualitative tendencies, we have performed the RPA calculation in relatively high temperature for various values of parameters to compare the

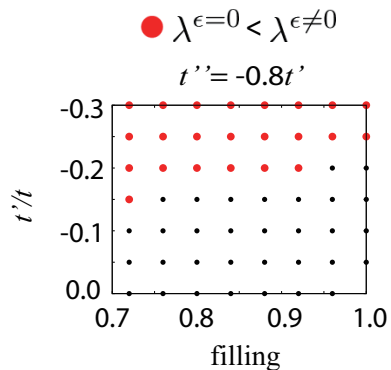


FIG. 5: (Color online) Comparison of the eigenvalue λ calculated within RPA between the four-fold symmetric Fermi surface ($\epsilon = 0$) and the distorted Fermi surface ($\epsilon = 0.01$) with red (black) dots representing the case of $\lambda^{\epsilon=0} < \lambda^{\epsilon=0.01}$ ($\lambda^{\epsilon=0} > \lambda^{\epsilon=0.01}$) for $\beta t = 5, U/t = 2.0$. The horizontal axis corresponds to the band filling from $n = 0.7$ to 1.0 , while the vertical axis is t'/t , with $t'' = -0.8t'$ (which includes the parameter set used in the Fig. 3).

four-fold symmetric case with the distorted Fermi surface by making the nearest-neighbor hopping anisotropic, $t_x = 1 + \epsilon, t_y = 1 - \epsilon$, by hand with the first line in Eq. (2) becoming $\epsilon(\mathbf{k}) = -2t_x \cos k_x - 2t_y \cos k_y$. In the RPA, we ignore the self-energy effect in the Eliashberg Eq. (6).

When we compare the eigenvalue under a distortion $\lambda^{\epsilon=0.01}$ with $\lambda^{\epsilon=0}$ for the symmetric case, the result in Fig. 5 for $U/t = 2.0, \beta t = 5$ shows that we do have a region (marked with red circles for $\lambda^{\epsilon=0.01} > \lambda^{\epsilon=0}$) in which the distortion enhances the eigenvalue. This effect tends to occur away from half-filling, and for larger values of distant-neighbor hopping t', t'' (i.e., more frustrated cases). Thus we can confirm that the enhancement of the superconductivity by small distortions indeed occurs in the weak-coupling limit where we can ignore the self-energy effect.

It has been known that the gap symmetry (for the leading eigenvalue) tends to be changed for higher doping or more frustrated cases³⁰. The present result suggests that the T_c^{SC} enhancement arising from the distortion tends to occur around the boundary for the gap symmetry to change where the leading and sub-leading eigenvalues are close to each other. This is also consistent with the above result for the t' dependence in FLEX+DMFT (Fig. 3), where the enhancement of T_c^{SC} occurs for $(t', t'') = (-0.20, 0.16)$. We also notice that the structure of Eq. (14) is reminiscent of the pseudo Jahn-Teller effect, in which a Jahn-Teller-like distortion occurs without degeneracies due to the second-order effect of the distortion³¹. In this context we can also recall a well-known property that, if the eigenvalues are degenerate (e.g. for $p + ip$ -pairing), T_c^{SC} can be enhanced by the strain effect³². From these, the present result may also be viewed as a possibility for this kind of T_c^{SC} enhancement revealed even for the (non-degenerate) d -wave regime in 2D square lattice Hubbard model.

IV. SUMMARY AND DISCUSSIONS

We have employed FLEX+DMFT approach to study the interplay of Pomeranchuk instability and superconductivity in correlated electron systems. We have revealed that the superconductivity with the distorted Fermi surface has the symmetry of the gap function changed from d -wave to $d + s$, consistent with the previous studies^{16,17}. We have found that the Fermi surface distortion can enhance the superconducting T_c^{SC} in the overdoped regime. We have explained this enhancement in terms of the perturbation for small distortions, and also with RPA in the weak-coupling regime. The Pomeranchuk T_c^{PI} is found to be much more sensitive to Fermi surface warping and the position of the van Hove filling than the superconducting transition temperature.

If we comment on the method, FLEX+DMFT, which is an improvement over FLEX or DMFT, still overestimates the non-local self-energy effect. For more accurate estimates, other methods (e.g. diagrammatic expansion in two-particle level as in DFA³³ or dual fermion method³⁴) will be needed. Also, we have assumed here translationally invariant systems, while the study of the interplay between superconductivity and charge instabilities involving finite wave vectors will be another interesting future work.

V. ACKNOWLEDGEMENT

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Appendix A: Effect of the DMFT vertex Γ_{DMFT}

According to the formulation in Ref.21, we should consider the local anomalous self-energy Δ_{loc} coming from the DMFT functional. Then the linearized Eliashberg equation becomes

$$\lambda \Delta(k) = -\frac{1}{N_k \beta} \sum_{k'} [V_{\text{eff}}(k - k') + \Gamma_{\text{DMFT}}(\omega_n, \omega_m)] \times |G(k')|^2 \Delta(k'), \quad (\text{A1})$$

where $\Gamma_{\text{DMFT}} = \delta \Delta_{\text{loc}} / \delta F$ is the functional derivative of the local anomalous self-energy. While this term can be ignored for studying pure d -wave pairing as in the previous paper²¹, we can examine its effect on the $d + s$ pairing. We consider this effect to be small, because the additional term is an extended s -wave (nonlocal) pairing rather than the ordinary s -wave, so that a cancellation should occur in the momentum summation. To check this

along the argument in the main text, we can calculate the lower bound of the maximal eigenvalue when Γ_{DMFT} is considered without calculating Γ_{DMFT} directly. From the eigenvector Δ_{max} of Eq. (6), we extract the part of the gap function that is not affected by Γ_{DMFT} as

$$\Delta'(k) = \Delta_{\text{max}}(k) - \frac{\sum_{\mathbf{k}} |G(k)|^2 \Delta_{\text{max}}(k)}{\sum_{\mathbf{k}} |G(k)|^2}. \quad (\text{A2})$$

Then a quantity,

$$\lambda' = - \frac{\sum_{k,k'} \Delta'^*(k) |G(k)|^2 V_{\text{eff}}(k-k') |G(k')|^2 \Delta'(k')}{\sum_k \Delta'^*(k) |G(k)|^2 \Delta(k)}, \quad (\text{A3})$$

gives the lower bound for the maximal eigenvalue when Γ_{DMFT} is considered. We have actually confirmed that the difference between λ' and λ (without Γ_{DMFT}) is very small, $(\lambda - \lambda')/\lambda < 0.01$. Thus we can conclude the effect of the DMFT vertex Γ_{DMFT} does not significantly change the the result for the T_c^{SC} enhancement.

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